Benha University Faculty of Engineering – Shoubra Department of Industrial Engineering Duration: 2 hours		Final Exam Course: Mathematics 3 Code: EMP 201 Date : December 23, 2017

The exam consists of one page | No. of questions: 4 | Answer **All** questions | Total Mark: 40

Question 1

(a)Find the first derivatives of the function :

$f(x,y,z) = x.\sin y + y\cos x + z\ln z$ — 2

(b)Find the envelope of the curves : $(x-\alpha)^2 + (y-\alpha)^2 = 4.$ — 4

(c)Determine the extrema of the function :

$\mathrm{f(x,y) = x + 3y}$ subject to $\mathrm{g(x,y) = x^2 + 3y^2 - 16 = 0}$ — 4

Question 2

(a) Find $\nabla.\overline{\mathrm{U}}$ and $\nabla \mathrm{x} \overline{\mathrm{U}}$ where : $\overline{\mathrm{U}} = (\mathrm{x^2 z})\mathrm{i} + (\mathrm{y^2 + z^3})\mathrm{j} + (\mathrm{x} + \sin \mathrm{z})\mathrm{k}.$ — 3

(b)Find the integral : $\iint_D \frac{y}{\sqrt{x^2+y^2}}\,dx\,dy$, D is $x^2 + y^2 = 4,\quad x,\ y \geq 0$ — 3

(c) Find the integral : $\int_{(0,0)}^{(1,2)} (x+y)dx + (2x-y)dy$ along the curve $y = 2x^2$ — 4

Question 3

(a)Find the Fourier series of the function :

$f(x) = \mathrm{x}\,,\quad -\pi \leq \mathrm{x} \leq \pi,\quad \mathrm{f(x+2\pi) = f(x)}$ — 5

(b)Find u and v of the function : $f(z) = \sin z$ and show that they satisfy Remman's equations. — 5

Question 4

(a)Find the integrals : $\oint_C \frac{z^3-5z}{(z-2)^3}dz$ where C is the circle | z | = 5. — 5

(b)Find the integral : $\int_{(3,0)}^{(-3,0)} \frac{2z^2-1}{z}dz$ on the upper half of $|z| = 3$. — 5

Good Luck *Dr. Mohamed Eid* *Dr. Fathi Abdsallam*

Model Answer

Answer of Question 1

(a) $f_x = \sin y - y \sin x,\quad f_y = x\cos y + \cos x,\quad f_z = z\,\frac{1}{z} + \ln z = 1 + \ln z$

--*2- Marks*

(b)Differentiate with respect to a, we get : $-2(x-\alpha) - 2(y-\alpha) \quad = 0.$

Then $\alpha = \frac{1}{2}(x+y)$. The envelope is : $(x-y)^2 = 8$

--*4- Marks*

(c)From : $x + 3y = \lambda(x^2 + 3y^2 - 16)$

Then $1 = \lambda(2x),\ 3 = \lambda(6y)$. Then we get $\lambda = \frac{1}{2x} = \frac{3}{6y}$. Then $y = x$

Substitute in g(x, y) : $x^2 + 3x^2 = 16$. Then $x^2 = 4$. Then $x = 2, -2$ and $y = 2, -2$

Then, we get the points (2, 2), (– 2, –2).

We see that f(2, 2) = 8 which is maximum and f(–2, –2) = – 8 which is minimum.

--*4- Marks*

Answer of Question 2

(a) $\nabla.\bar{U} = 2xy + 2y + x + \cos z.$

$$\nabla x\bar{U} = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x^2 z & y^2 + z^3 & x + \sin z \end{vmatrix} = (-3z^2)i - (1 - x^2)j$$

--*3- Marks*

(b) $$I = \int_0^{\frac{\pi}{4}} \int_0^2 \frac{r\sin\theta}{r}\ r\,dr\,d\theta = \int_0^{\frac{\pi}{4}} \int_0^2 r\sin\theta\ dr\,d\theta = 2\int_0^{\frac{\pi}{4}} \sin\theta\ d\theta = 2 - \frac{2}{\sqrt{2}}$$

--*3- Marks*

(c) From $y = 2x^2,\ dy = 4x\,dx$. Then

$$I = \int_0^1 (x + 10x^2 - 8x^3)\,dx = \frac{11}{6}$$

--*4- Marks*

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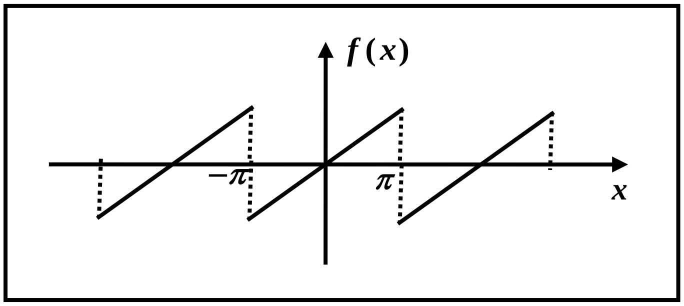
Answer of Question 3

(a) $f(x) = x \qquad -\pi < x < \pi$



$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\,dx = \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{\pi} = \frac{1}{4\pi}\left[\pi^2 - \pi^2\right] = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos n\pi x\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos nx\,dx = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin nx\,dx = \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\,dx$$

$$= \frac{2}{\pi}\left[\frac{-x}{n}\cos nx + \frac{1}{n^2}\sin nx\right]_0^{\pi} = \frac{2}{\pi}\left[\frac{-\pi}{n}\cos n\pi\right] = \frac{-2}{n}(-1)^n = \frac{2}{n}(-1)^{n-1}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n\cos nx + b_n\sin nx\right] \qquad \boxed{x = 2\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\sin nx}$$

--5- Mark

(b)Show that $w = \sin z$ satisfy Cauchy Remman's Equation.

$$w = \sin z = \sin(x+iy) = \sin x\cos iy + \cos x\sin iy = \sin x\cosh y + i\cos x\sinh y$$

$$u = \sin x\cosh y \qquad v = \cos x\sinh y$$

$$\frac{\partial u}{\partial x} = \cos x\cosh y \qquad \frac{\partial v}{\partial x} = -\sin x\sinh y$$

$$\frac{\partial u}{\partial y} = \sin x\sinh y \qquad \frac{\partial v}{\partial y} = \cos x\cosh y$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = \cos x\cosh y \qquad and \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} = \sin x\sinh y$$

Then $w = \sin z$ satisfy Cauchy Remman's Equation.

---5-Marks

Answer of Question 4

(a) $\dfrac{z^3-5z}{(z-2)^3}$ is analytic except at $z=2$ inside $|z|=5$ use Cauchy integral theorem

Then $a=2$ and $f(z)=z^3-5z$ Which analytic inside and on $|z|=5$

$f'(z)=3z^2-5, f''(z)=6z$ and $f''(a)=6(2)=12$

$$\oint_C \frac{z^3-5z}{(z-2)^3}dz=\frac{2\pi i\ f''(a)}{2!}=\frac{2\pi i\ (12)}{2}=\boxed{12\pi i}$$

---5- Marks

(b)Evaluate $\displaystyle\int_{(3,0)}^{(-3,0)} \frac{(2z^2-1)}{z}dz$ on the upper half of the circle $|z|=3$ Answer

Put $z=3e^{i\theta}$ *then* $dz=3ie^{i\theta}d\theta$

$$\int_{(3,0)}^{(-3,0)} \frac{(2z^2-1)}{z}dz=\int_0^{\pi}\frac{(18e^{2i\theta}-1)}{3e^{i\theta}}3ie^{i\theta}d\theta=i\int_0^{\pi}(18e^{2i\theta}-1)d\theta=i\left[\frac{18e^{2i\theta}}{2i}-\theta\right]_0^{\pi}$$

$$=\left[9e^{2i\theta}-i\theta\right]_0^{\pi}=\left[9e^{2\pi i}-9e^0-i\pi\right]=-\pi i$$

---*5-Marks*

Dr. Fathi Abdsallam